Stochastic and spectral analysis of time series for identifying periodic land use

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Abstract

This application note shows the interest of the time series auto / cross covariance feature to mine periodic land use / land cover (`LU/LC`) in an agricultural landscape. A Bayesian point of view has been adopted to cope with the variability of the real-world data sets. The agricultural landscape is represented by a mosaic of plots, each of them holding a time series of annual `LU/LC` surveys. An agricultural district is seen as a sample drawn from a time series random process. For each `LU`, the autocovariance function of the random process is first calculated and its Fast Fourier Transform (FFT) performed. The peaks of the power spectrum determine the frequencies – or the corresponding periods – of this `LU`. In addition, the analysis of the crosscovariance between two `LU/LC` may exhibit the co-occurrences of these `LU/LC` at different lags and determine a return time.

In the data mining of agricultural landscapes represented by a mosaic of agricultural plots, results on time series coming from annual `LU/LC` surveys

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are presented. They show that the auto / cross covariance coefficients and their spectral representation give immediate and accurate information to a data mining analyst on complex land use rotations and their trends for understanding the underlying logical process in landscape dynamics.

Keywords:
Periodic crop, land use succession, multivariate random process, time series analysis, autocovariance

1. Introduction

In agricultural landscapes, the time series of land use and land cover (`LU/LC`) seem randomly distributed among different agricultural fields (plots) managed by farmers. Nevertheless, they reveal the presence of logical processes and driving forces related to the soil, climate, cropping system, and economical pressure whose understanding is a major challenge mainly for landscape agronomists (Benoît et al., 2012). In these time series, the observation of periodic behavior is of interest for achieving a concise data set representation and extracting knowledge about the underlying logical process. We propose in this application a stochastic framework in which stationary time series of `LU/LC` coming from annual surveys are analyzed to look for periodic `LU` and determine crop rotations. A Bayesian point of view has been adopted to cope with the variability of the real-world data sets. The data are assumed to come from a probabilistic source that produces synchronous categorical data. Each agricultural district is seen as a sample drawn from a time series random process.

The originality of our work is twofold: on the one hand, following Berberidis et al. (2002); Elfeky et al. (2005); Geis and Einax (1996), we propose a method to process time series by means of a spectral analysis of autocovariance coefficients, but we extent their work by using crosscovariance coefficients and categorical data sets. On the other hand, several series of items are handled simultaneously considering them as samples from a random source of categorical variables whose stochastic moments are efficiently estimated when the ergodicity property stands.

We extent our preliminary study on the modeling of the spatial distribution of crop sequences in France (Xiao et al., 2014) by showing on two specific agricultural districts how the autocovariance coefficients and their spectral representation can discern the `LU` rotations and their trends even in a complex succession scheme.

2. Stochastic framework

Let's consider a time sequence $x_1, x_2, \dots x_T$ of T `LU/LC` drawn from a set $\mathcal{E} = \{e_1, e_2, \dots e_K\}$ of K different labels observed at time $1, 2, \dots T$ at a particular plot in an agricultural territory. The x_t are the realizations of a random variable $X_t(\omega)$ at time t in a particular plot represented by ω.

Suppose we have logged the `LU/LC` of a mosaic of plots. The data set defines a matrix M where row ω represents the T `LU/LC` of plot ω observed at year $1, 2, \dots T$. Column t represents the `LU/LC` seen in the whole territory at time t. This matrix represents a sampling set of the time series $X_1(\omega), X_2(\omega), \dots X_T(\omega)$ viewed as a random process.

2.1. Autocovariance Function

In discrete stationary random process, the autocovariance function between labels e_i and e_j is defined as:

$$R_{XX}(\tau)_{[i,j]} = \frac{1}{T}\sum_t Prob(x_t^i, x_{t+\tau}^j) - E^i E^j \quad (1)$$

when x_t^i means that the `LU/LC` at time t is e_i and E^i is the mean of label e_i in the data set M thanks to the ergodicity property.

The probability $Prob(x_t^i, x_{t+\tau}^j)$ can be estimated by counting in data set M the rows where the couple (e_i, e_j) is seen in columns t and $t+\tau$ respectively.

$R_{XX}(\tau)_{[i,j]}$ provides information on co-occurrences of e_i and e_j at different lags expressed by τ. It can be used as a feature to hypothesize symbol successions from periodic noisy time series. The result of the spectral analysis by means of a FFT (Priestley, 1981) of $R_{XX}(\tau)_{[i,j]}$ as a function of τ is a periodogram whose peaks determine the periods of e_i (see Fig.2(b)).

3. Analyzing the return time between crops and meadows in the Vittel area

The data used in this study were obtained within the framework of the research project AGREV 3 (Agriculture Environment Vittel) which accompanies Agrivair - a subsidiary of *Nestle water* - in its actions of protection of natural mineral water. To reach this goal , it is necessary to set up and to assess agricultural practices which provide a long-term protection of the resources. As a consequence, crop rotations must be attentively monitored and *Nestle water* encourages farmers to comply with specifications in which aid is granted to encourage planting soils in temporary meadows and a crop management without pesticide.

The plot mosaic of the watershed of Vittel Contrexéville was annually investigated between 1996 and 2011. Each parcel has a modality among 11 labels. A time-space clustering of this mosaic has been first performed on the basis on `LU/LC` successions to specify a sub-territory where the `LU/LC` successions are modeled by a stationary hidden Markov model by ARPEnT-Age (Mari et al., 2013). The cross covariance was next performed and gave new valuable information on long-term successions by discovering interesting co-occurrences of `LU/LC` at different lags as shown in Fig. 1.

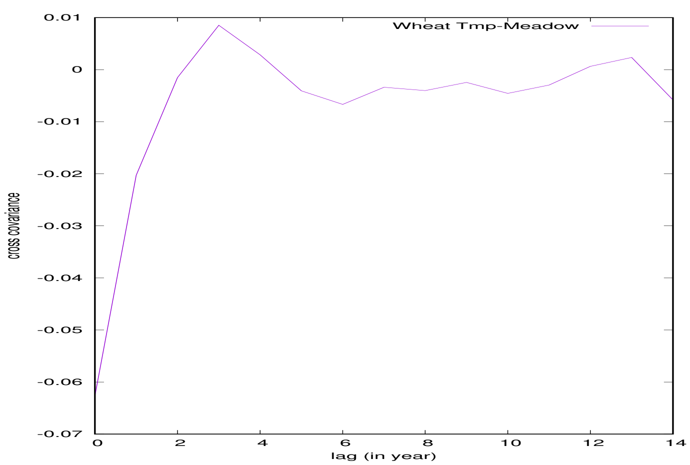


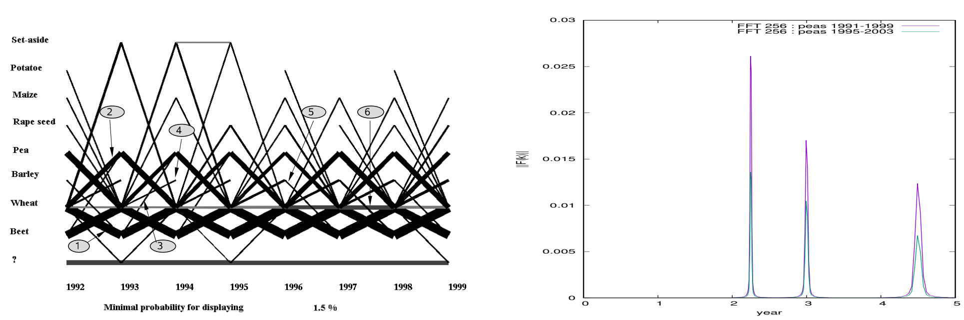
Figure 1: Cross covariance between "Wheat" and "Temporary Meadow" at different lags. The peak located at $\tau = 3$ is related to the 4-year succession [("Crop" x 3)-"Temp. Meadow"]. The drop located near $\tau = 6$ means that a temporary meadow cannot be 6 years later because this `LU/LC` cannot last more than 5 years

Figure 1 shows an administrative and agronomic feature of the parcels that hold grain crops (Wheat or Barley) in a sub-territory mainly developed by under-contract farmers. In this sub-territory – represented by 42598 16-

length time series – a parcel cannot hold a crop for more than 3 years due to the no-pesticide policy. Therefore a sequence of temporary meadows must be inserted in the mono-culture of crop. This can explain the peak at $\tau = 3$ because the succession ["Crop" x 3)-"Temp. Meadow"] is frequent. But the "Temp. Meadow" cannot last more than 5 years, otherwise this parcel becomes a permanent meadow and cannot hold another `LU`. Therefore, before this deadline, these parcels hold another more economically interesting `LU`, like grain crops. This explains the drop at lag $\tau = 6$ in the cross covariance function.

4. Analyzing crop sequences in the Saint-Quentinois

For thirty or forty years, increasing human activities (domestic, industrial, agricultural) have gradually degraded the hydro-system of the Seine river, regarding water quality and biological populations.



(a) Crop transitions

(b) Main return time of "Peas"

Figure 2: Crop transitions between 1992 and 1999 in the district of Saint-Quentinois (North-east of France) (after Le Ber et al. (2006); Xiao et al. (2014)). The FFT analysis of the autocovariance of "peas" shows 3 periods and a decline of this `LU` between [1991-1999] and [1995-2003]

In this analysis, the autocovariance function and its FFT have been performed on 2336 time series of annual surveys collected between 1991 and 2003. Each `LU/LC` has been classified into eleven modalities. The spectral feature can highlight the periods of `LU` involved in complex rotation scheme. The main annual transitions between crops and their trends are displayed in figure 2(a) (Le Ber et al., 2006)). The importance of the transition between two crops is expressed by the thickness of the line joining the two crops. In figure 2(a), the 3-year successions "Wheat-Beet-Wheat" (1) and "Pea-Wheat-Pea" (2) seem predominant but no information on longer successions is given. A spectral analysis by means of a FFT on the autocovariance of "Pea" (Fig. 2(b)) gave a periodogram that immediately showed that this `LU` had 3 periods that can be rounded to: 2, 3, 4 between 1992 and 1999. A similar analysis on the time series collected between 1995-2003 showed a periodogram with less magnitude indicating a decline of this `LU`. These informations have already been given by Le Ber et al. (2006) who performed a long and tedious analysis of the triple and quadruple `LU` in which "Pea" was involved. The rotations: "Pea-Wheat", "Beet-Wheat-Pea" and finally "Beet-Wheat-Pea-Wheat" were revealed at that time.

5. Discussions and conclusions

Considering all the `LU/LC` sequences as a sampling set of a time series of random variables $X_t(\omega)$, the autocovariance function can be estimated efficiently and detects the `LU/LC` periodicity. In addition, the co-occurrence of two `LU/LC` at different lags can be detected by the cross covariance. In the field of agricultural landscapes, this eases the work of an analyst in retrieving periodic sequences or long-term successions for understanding the underlying logical process in landscape dynamics.

In these experiments, we have assumed that the data have two main properties: stationarity and ergodicity. Stationarity means that the moments of a time series do not change over the time. In our application, as far as the driving forces encountered by a farmer on a particular plot do not change, stationarity of the LU/LC time series can be assumed. Ergodicity is a less known property that states that the ensemble averages can replace the time averages. Ergodicity can be assumed in an agricultural landscape where the crop distribution in the landscape is time independent and corresponds to the crop distribution in a plot. Then, the time and space delimitation of the data must be carefully chosen.

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Appendix A. Spectral analysis of the autocovariance function

Let's consider a time sequence $x_1, x_2, \ldots x_T$ of T items drawn from a set $\mathcal{E} = \{e_1, e_2, \ldots e_K\}$ of K different labels like the time series made up with T land use (`LU`) observed at time $1, 2, \ldots T$ at a particular plot in an agricultural territory. Each `LU/LC` belongs to the set $\mathcal{E}$. The x_t are the realizations of a random variable $X_t(\omega)$ at time t in a particular plot represented by ω. The time series at a specific plot ω is represented by a sequence of T vectors belonging to $\mathcal{R}^K$. Vector x with all components 0 except the i-th being 1 will represent item e_i. Component i of vector x_t is the 1 function – also called the Dirac function – that represents the observation of e_i as a function of t. The time series of items are then parameterized by a series of multivariate 1 functions allowing processing by the techniques of multivariate random signals.

When the joint probability distribution $Prob(x_t, x_{t+\tau})$ is known, the auto-/ cross second moment is defined by the expectation (Priestley, 1981):

$$\begin{aligned} C_X(\tau) &= E\left[X_t(\omega)X^*_{t+\tau}(\omega)\right] \\ &= \frac{1}{T}\sum_t x_t x^*_{t+\tau} Prob(x_t, x_{t+\tau}) \end{aligned} \tag{A.1}$$

where x^* represents the transpose of vector x.

Each term $x_t x^*_{t+\tau}$ in the sum of Eq. A.1 is a $K \times K$ matrix. When $(x_t, x_{t+\tau})$ represents labels (e_i, e_j) at time $(t, t+\tau)$ the product $x_t x^*_{t+\tau}$ is a matrix with 1 at index $[i, j]$ and 0 everywhere else.

Therefore, the general term $[i, j]$ of $C_X(\tau)$ is:

$$C_X(\tau)[i,j] = \frac{1}{T}\sum_t Prob(x^i_t, x^j_{t+\tau}) \tag{A.2}$$

where x_t^i is the component i of vector x_t and means that item e_i has been observed at time t.

On the basis of Equation A.1, the autocovariance function is defined as an auto cross second moment with centered random variables.

$$R_{XX}(\tau) = E\left[X_t(\omega)X^*_{t+\tau}(\omega)\right] - E\left[X_t(\omega)\right]E\left[X^*_{t+\tau}(\omega)\right] \tag{A.3}$$

If a stochastic process is stationary, these statistic moments are time invariant. In that case:

$$E\left[X_t(\omega)\right] = E\left[X_{t+\tau}(\omega)\right] = E\left[X(\omega)\right]$$

The general term $[i,j]$ is equal to:

$$R_{XX}(\tau)[i,j] \quad = \quad \frac{1}{T}\sum_t Prob(x_t^i, x_{t+\tau}^j) - E^iE^j \tag{A.4}$$

where E^i is component i of vector $E\left[X(\omega)\right]$.

The frequencies that compose the autocovariance function $R_{XX}(\tau)[i,j]$ as a function of τ may be extracted by a discrete Fourier transform (FFT for real valued function or DFT in the case of discrete data).

$R_{XX}(\tau)[i,j]$ is first windowed with a N-length Hamming window to alleviate the artifacts related to the limited nature of the sequence.

Let's denote $f_{i,j}(\tau) = R_{XX}(\tau)[i,j]$ a function of τ whose parameters are i and j, the DFT $\mathcal{F}_{i,j}(k)$ on N points is defined by:

$$\mathcal{F}_{i,j}(k) = \sum_{\tau=0}^{N-1} f_{i,j}(\tau)\exp -j\frac{2\pi k\tau}{N} \quad k = 0, \cdots N-1 \tag{A.5}$$

the values $\frac{k}{N}$ are called the frequencies whereas $\frac{N}{k}$ represents the periods. Let's denote $\parallel \mathcal{F}_{i,j}(k) \parallel$ the modulus of the complex number $\mathcal{F}_{i,j}(k)$. The values $\parallel \mathcal{F}_{i,j}(k) \parallel^2, \quad k = 0, \frac{N-1}{2}$ define the power spectrum.

The graphical representation of the power spectrum as a function of k/N is called a spectrogram and viewed as a function of N/k a periodogram.

To get a reasonable frequency resolution, it is possible to use longer windows that are padded with zero. In our work, due to the limited length of the sequences (roughly between 10 and 20 items), the sequences to fill the DFT window have been duplicated and padded with zeros.

If the hypothesis of a stationary process is unrealistic, it is possible to define a narrower sliding window that will be shifted along the data set.